CALCULATION OF THE THERMAL FIELD OF THE THROTTLE ELEMENT OF APPARATUS FOR STUDYING THE JOULE—THOMSON EFFECT

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A method of designing the throttle element of apparatus for studying the Joule—Thomson effect taking into account the thermal conductivity is proposed.

In different laboratory devices for determining the Joule—Thomson coefficient [1-3] the thermal losses are eliminated by insulating the throttling element. This considerably complicates the device. In addition, for small velocities of motion of the fluid being investigated, even when there is thermal insulation, the effect of thermal conductivity on the thermal field of the throttling liquid is considerable. In this paper we show that in this case by choosing an appropriate method of interpretation and the dimensions of the throttling element one can considerably reduce the error in determining the Joule—Thomson coefficient due to heat losses.

The problem describing the thermal field of the Joule—Thomson effect in a throttling element taking into account the thermal conduction along the throttling and heat transfer paths through the side surface of the throttle is difficult to solve, so we will consider the contribution of these processes separately. This assumption is generally accepted when calculating temperature fields.

1. The Effect of the Thermal Conductivity of the Medium Along the Throttling Path of the Fluid on the Thermal Field of the Throttling Element. The mathematical formulation of the problem in the one-dimensional case has the form

$$a\frac{\partial^2 T(x,t)}{\partial x^2}-u(x,t)\left[\frac{\partial T(x,t)}{\partial x}+\varepsilon\frac{\partial P(x,t)}{\partial x}\right]=\frac{\partial T(x,t)}{\partial t};\quad t>0;\ -\infty<x<\infty; \tag{1}$$

$$T(x,0)=0. \tag{2}$$

The rate of convective heat transfer u(x, t) is related to the filtering speed of the fluid v(x, t) by the relation

$$u(x,t)=\frac{C_f}{C_p}v(x,t),$$

and the relation between v(x, t) and the pressure is given by Darcy's law

$$v(x,t)=-\frac{k}{\mu}\frac{\partial P(x,t)}{\partial x}.$$

We will further assume that the fluid flows freely from infinity to the point x = 0 and flows away from the point x = L under the action of a small pressure drop, while the main pressure drop is localized in the region 0 < x < L. In this case the pressure can be described approximately by the relation

$$P(x)=\begin{cases}0, & x<0;\\ -\dfrac{\Delta P_0}{L}x, & 0\leqslant x\leqslant L;\\ -\Delta P_0, & x>L,\end{cases} \tag{3}$$

while the rate of convective heat transfer u = const.

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The solution of problem (1)-(2) can easily be obtained by using the theory of generalized functions. Assuming that the pressure profile P(x) is connected at the instant of time t = 0, the solution can be written in the form

$$T(x, t) = -\frac{\varepsilon u}{2\sqrt{\pi a}} \int_0^t dt' \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{[x-x'-u(t-t')]^2}{4a(t-t')}\right\}}{\sqrt{t-t'}} \frac{\partial P(x')}{\partial x'} dx'. \tag{4}$$

It is easy to prove the correctness of Eq. (4) by substituting it directly into (1).

Further, taking into account the fact that

$$\lim_{\beta\to 0} \frac{1}{\beta} \exp\left(-\frac{\alpha^2}{\beta^2}\right) = \sqrt{\pi}\,\delta(\alpha),$$

we have

$$T(x, t)|_{a\to 0} = -\varepsilon u \int_0^t \frac{\partial P[x-u(t-t')]}{\partial x} dt' = -\varepsilon[P(x) - P(x-ut)]. \tag{5}$$

Expression (5) is the same as the solution of (1)-(2) obtained ignoring thermal conductivity [4].

Substituting (3) into (4) we obtain

$$T = \frac{\varepsilon u \Delta P_0}{2L} \int_0^t \left\{\operatorname{erf}\left[\frac{x-L-u(t-t')}{2\sqrt{a(t-t')}}\right] - \operatorname{erf}\left[\frac{x-u(t-t')}{2\sqrt{a(t-t')}}\right]\right\} dt'.$$

The relative temperature $T/\varepsilon\Delta P_0$ at the point x = L (i.e., at the output of the porous element of the throttle) will vary with time as given by

$$\frac{T(L, \mathrm{Fo})}{\varepsilon\Delta P_0} = k\,\mathrm{Fo} \int_0^1 \left\{\operatorname{erf}\left[\frac{1-2k\,\mathrm{Fo}(1-x)}{2\sqrt{\mathrm{Fo}(1-x)}}\right] + \operatorname{erf}\left[k\sqrt{\mathrm{Fo}(1-x)}\right]\right\} dx. \tag{6}$$

Here $\mathrm{Fo} = at/L^2$ is the Fourier number and $k = uL/2a$ is the convection parameter.

It follows from Eq. (5), in deriving which we ignored the thermal conductivity of the medium, that

$$\frac{T(L, \mathrm{Fo})}{\varepsilon\Delta P_0} = 2k\,\mathrm{Fo}\,\gamma\left(\mathrm{Fo} \leqslant \frac{1}{2k}\right) + \gamma\left(\mathrm{Fo} > \frac{1}{2k}\right), \tag{7}$$

where

$$\gamma(a<b) = \begin{cases} 1, & \text{if } a<b; \\ 0, & \text{if } a>b. \end{cases}$$

It follows from (7) that the temperature at the output of the porous element varies linearly up to its maximum value $\Delta T_m = \varepsilon\Delta P_0$ in a time $t_0 = L/u$ and then remains unchanged.

Figure 1 shows the results of computer calculations of the relative temperature x = L as a function of log Fo using Eqs. (6) and (7).

Using the curves shown in Fig. 1, assigning actual possible values to the time that the experiment takes for known values of the fluid flow Q and the thermal properties of the porous element, we can choose the optimum length of the throttling element L for which the temperature of the throttling fluid at the output of the throttling element has its maximum value. For example, the apparatus enables one to maintain the rate of flow of the fluid constant and equal to 5 m/sec. Further, suppose the thermal properties of the throttling element are such that u = 2 cm/sec, and the duration of the experiment is limited by the particular features of the apparatus (the volume of the working fluid) and is 60 sec. With this data the optimum length of the throttling element is not greater than 120 cm, otherwise the temperature of the fluid at the output of the throttling element after 60 sec cannot be established. The results of calculations for known parameters of the throttle element can be used to choose the optimum values of the flow of the fluid being investigated and the time taken to carry out filtering before recording the temperature change.

2. Calculation of the Effect of Heat Transfer through the Side Surface of the Throttle on the Thermal Field of the Throttling Fluid. Suppose conditions are arranged, i.e., the optimum values of the length of the throttle element L and the fluid flow are chosen taking its thermal properties into account, so that at the output of the throttle element after a time t the effect of the thermal conductivity along the fluid throttling path of the measured temperature can be neglected. However, when constructing throttle elements to study the Joule–Thomson effect it is impossible to ensure thermal insulation of the throttle surface, and hence when throttling a fluid heat losses will be inevitable. If the contribution of heat transfer through the surface of the throttle element into the surrounding medium on the formation of the thermal field of the throttling liquid is ignored, considerable errors may arise, particularly for low filtering speeds and small radii of the transverse cross section of the throttle element.

We will assume that the heat exchange between the throttle element and the surrounding medium at constant temperature obeys Newton's law, while the temperature of the fluid passing through the throttle element is the same as the temperature of the surrounding medium.

The mathematical formulation of the problem has the following form:

$$a\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)-u\left[\frac{\partial T}{\partial x}+\varepsilon\frac{\partial P}{\partial x}\right]=\frac{\partial T}{\partial t};\quad 0\leqslant r<R;\ x,\ t>0; \tag{8}$$

$$T|_{t=0}=0;\ T|_{x=0}=0; \tag{9}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R}+hT|_{r=R}=0. \tag{10}$$

As can be seen from Eqs. (9)-(10) in this case, T(r, x, t) represents the difference between the temperature of the fluid at the point x, r at the instant of time t and the temperature of the fluid applied to the throttle element.

The solution of Eqs. (8)-(10) obtained by an operational Laplace transformation with respect to time and a finite Hankel transformation with respect to the variable r, has the form

$$T(x,r,t)=2\varepsilon\sum_{n=1}^{\infty}\frac{\mu_n J_0\left(\mu_n\frac{r}{R}\right)J_1(\mu_n)}{[\mu_n^2+\mathrm{Bi}^2]J_0^2(\mu_n)}\int_0^x\frac{\partial P}{\partial y}\exp\left[-\frac{a\mu_n^2}{uR^2}(x-y)\right]\gamma\left(t>\frac{x-y}{u}\right)dy, \tag{11}$$

where $J_0(x)$ and $J_1(x)$ are Bessel functions of a real variable, Bi = hR is the Biot criterion, and $\mu_n$ are the roots of the transcendental equation $\mathrm{Bi}J_0(\mu_n)=\mu_nJ_1(\mu_n)$. It is easy to show that solution (11) satisfies Eq. (8) and the boundary conditions (9) and (10).

For the case when Bi → ∞, i.e., when a constant temperature is maintained on the side surface of the throttle element, equal to the temperature of the applied fluid at x = 0, as in [3], Eq. (11) becomes

$$T(x,r,t)=2\varepsilon\sum_{n=1}^{\infty}\frac{J_0\left(\mu_n\frac{r}{R}\right)}{\mu_nJ_1(\mu_n)}\int_0^x\frac{\partial P}{\partial y}\exp\left[-\frac{a\mu_n^2}{uR^2}(x-y)\right]\gamma\left(t>\frac{x-y}{u}\right)dy, \tag{12}$$

where $\mu_n$ are the roots of the equation $J_0(\mu_n)=0$.

For a linear pressure distribution P(x) taking (3) into account when calculating the relative temperature $T/\varepsilon\Delta P_0$ for x = L, i.e., at the output of the throttle element, we have two solutions:

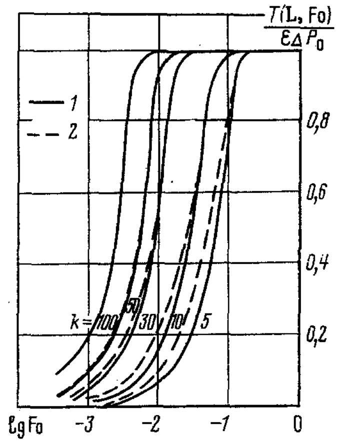


Fig. 1

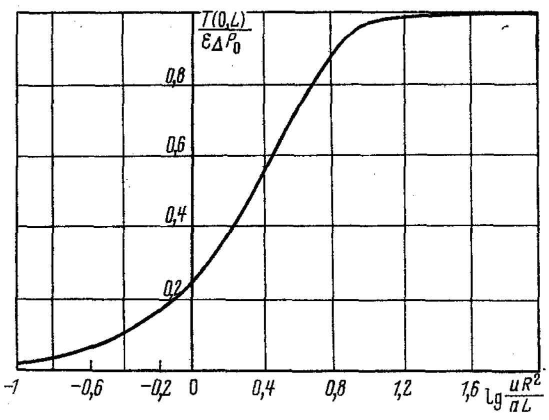


Fig. 2

Fig. 1. Dependence of the relative temperature $T(L, Fo)/\varepsilon\Delta P_0$ on log Fo for different $k = uL/2a$: 1 and 2 are when taking into account and ignoring the thermal conductivity, respectively, using Eqs. (6) and (7).

Fig. 2. Relative temperature at the output of the throttle element $T(L, 0, Fo)/\varepsilon\Delta P_0$ as a function of $\log(uR^2/La)$.

1) for $t < L/u$, i.e., for the instant of time when the liquid, applied at the instant of time $t = 0$ and $x = 0$, has not yet reached the point $x = L$;

$$\frac{T(L, r, t)}{\varepsilon\Delta P_0} = -\frac{2uR^2}{La}\sum_{n=1}^{\infty}\frac{J_0\left(\mu_n\frac{r}{R}\right)}{\mu_n^3 J_1(\mu_n)}\left[1-\exp\left(-\mu_n^2\frac{at}{R^2}\right)\right]; \tag{13}$$

2) for $t > L/u$

$$\frac{T(L, r, t)}{\varepsilon\Delta P_0} = -2\frac{uR^2}{La}\sum_{n=1}^{\infty}\frac{J_0\left(\mu_n\frac{r}{R}\right)}{\mu_n^3 J_1(\mu_n)}\left[1-\exp\left(-\frac{a\mu_n^2}{uR^2}L\right)\right]. \tag{14}$$

As can be seen from Eqs. (13) and (14) the establishment of a thermal field in this case occurs at the rate of convective heat transfer, and the maximum steady-state value of the relative temperature at the output of the porous element is described by Eq. (14). It follows from this that the change in temperature of the fluid due to the throttle effect is a minimum when $r = R$, since constant temperature is maintained on the surface, and is a maximum on the axis of the throttle element at maximum distance from the surface, so that when carrying out the experiments it is advisable to place temperature sensors on the axis.

The relative steady-state change in the temperature of the fluid at the output of the throttle element can be calculated from the equation

$$\frac{T(L, 0, \mathrm{Fo})}{\varepsilon\Delta P_0} = 2b\sum_{n=1}^{\infty}\frac{1-\exp\left(-\frac{\mu_n^2}{b}\right)}{\mu_n^3 J_1(\mu_n)}, \tag{15}$$

where $b = uR^2/La$.

Figure 2 shows $T(0, L, Fo)/\varepsilon\Delta P_0$ as a function of the logarithm of the parameter $uR^2/La$. As might have been expected, as the parameter b increases, i.e., as the velocity of fluid filtering or the radius of the throttle element is increased and the length and thermal conductivity of the throttle element is reduced, the measured temperature approaches its maximum value. For example, for $uR^2/La = 15.8$ the steady-state temperature at the output of the throttle element on the axis $r = 0$ can be assumed to be equal to its maximum value $\varepsilon\Delta P_0$. For

L = 1 m, u = 2 cm/sec, and $a = 2 \cdot 10^{-3}$ m²/h (dense sandstone) the radius of the throttle element should not be less than 2 cm. The maximum change in temperature in this case becomes established not earlier than one minute. For a rate of convective heat transfer of 10 cm/min, to observe the maximum change in temperature on the axis at the output of a throttle element 100 cm long it is necessary to choose the radius to be not less than 8 cm.

For small rates of flow of the fluid the value of the steady temperature on the axis of the throttle element (Fig. 2) depends on the velocity of motion of the fluid and the dimensions of the throttle element. In this case the Joule–Thomson coefficient must be calculated from the equation

$$\varepsilon = \frac{\Delta T}{A(b)\,\Delta P},$$

where the correction coefficient

$$A(b) = 2b \sum_{n=1}^{\infty} \frac{1 - \exp\left(-\frac{\mu_n^2}{b}\right)}{\mu_n^3 J_1(\mu_n)}.$$

This coefficient can be found from Fig. 2 for known parameters of the throttle element and the filtering of the fluid.

Hence, the theory and results given here enable the experimental method and the interpretation of the results to be improved when determining the Joule–Thomson coefficient. The need to do so arises due to the extensive use of thermometry when investigating cores and strata.

## NOTATION

T, temperature; P, pressure distribution; x, r, coordinates; t, time; $a$, thermal conductivity; u, rate of convective heat transfer; ε, Joule–Thomson coefficient; R, L, radius and length of the porous elements, respectively; h, heat-transfer coefficient; $C_f$, $C_p$, heat capacity of the fluid and porous medium, respectively; $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-u^2)\,du$.

## LITERATURE CITED


1. G. F. Trebin, Yu. V. Kapyrin, and V. N. Petukhov, "Experimental study of the measurement of temperature when throttling petroleum," in: Nauchnykh Trudov VNIIneft. No. 49, 74–80 (1974).
2. A. B. Marktovich, "Apparatus for investigating the thermodynamic properties of stratified petroleum deposits," Neftyanik, No. 8 (1966).
3. R. Ésedullaev, "Experimental investigation of the throttle effect when filtering gas in a porous medium," Uch. Zap. Azerb. Inst. Nefti Khim., Ser. 9, No. 8, 32 (1976).
4. É. B. Chekalyuk, Thermodynamics of Petroleum Layers [in Russian], Nedra, Moscow (1965), p. 51.